DE LA RECHERCHE À L'INDUSTRIE

Eigenvalue Problems in Three-dimensional Random Media

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Workshop MaNu |

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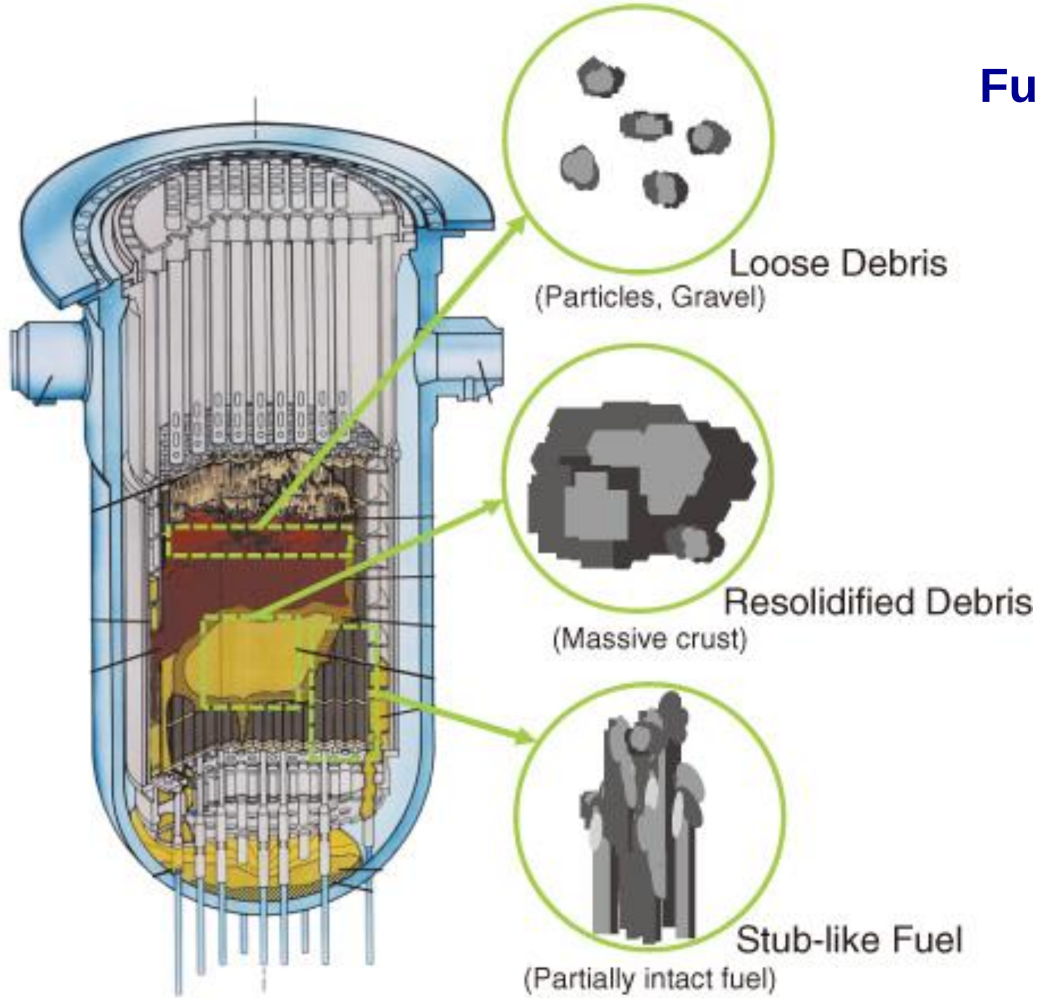
OUTLINE

- ❑ Eigenvalue problems in random media
- ❑ Stochastic tessellations
- ❑ Analysis of fuel assemblies
- ❑ Perspectives

CONTEXT AND MOTIVATIONS

Analysis of **re-criticality** probability following **severe core accidents**



Fuel degradation:

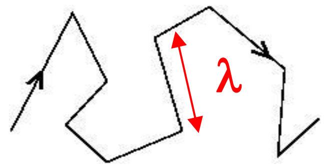
- melting and solidification
- random configurations

TMI corium sample

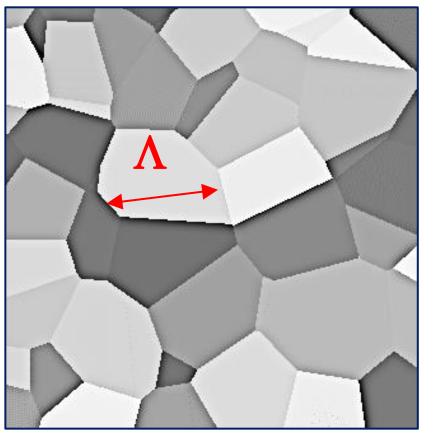
NEUTRON TRANSPORT IN « RANDOM » MEDIA

The **randomness** of a medium is measured w.r.t. **neutron transport**

Define :



- ❑ $\lambda = 1 / \Sigma =$ neutron mean free path
- ❑ $\Lambda =$ typical disorder size (*correlation length*) :
  average size of homogeneous regions



Three possible **regimes** :

$\Lambda \ll \lambda$ | $\Lambda \approx \lambda$ | $\Lambda \gg \lambda$ → Λ

- **Microscopically homogeneous** medium (« ***atomic mix*** »)

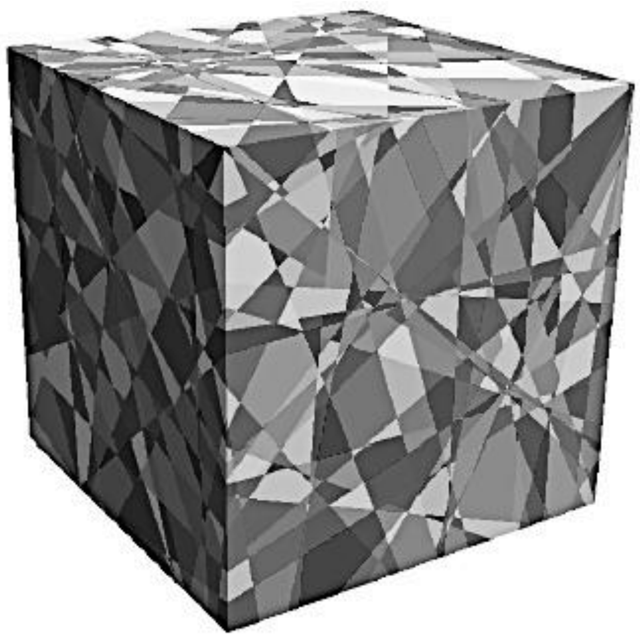
- **Microscopically heterogeneous** medium
- Disorder can be described by **probabilistic models**

- **Macroscopically heterogeneous** medium (homogeneous « by blocks »)

STOCHASTIC TESSELLATIONS FOR CRITICALITY PROBLEMS

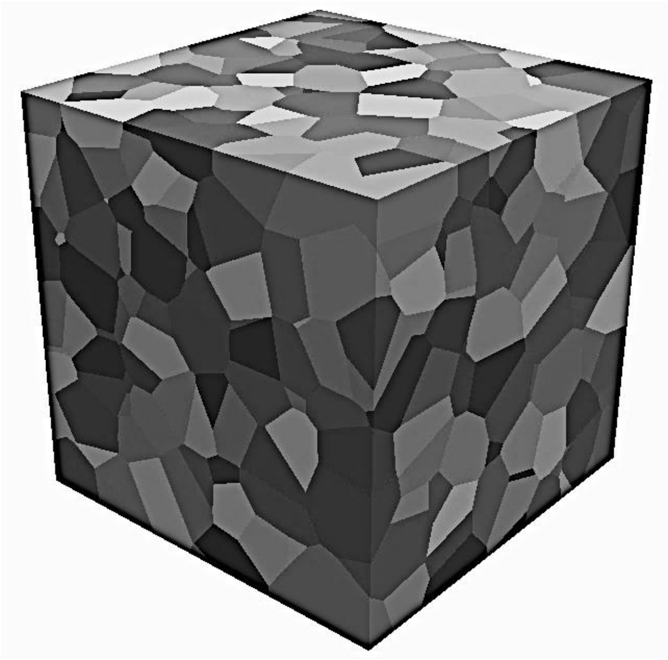
Stochastic tessellations [Miles, Santalo] are a convenient model for disorder

- Idea: partition a d-dimensional region according to a given probability law *P(q)*



Poisson tessellations

- Random hyper-planes
- Markov property



Voronoi tessellations

- Poisson point process
- Voronoi diagrams

Eigenvalue Boltzmann equation for the neutron flux φ :

For a single geometry realization *q*

$$\underbrace{\boldsymbol{\omega}\cdot\nabla\varphi_k^{(q)}}_{\text{Leakage}}+\underbrace{\Sigma_t^{(q)}\varphi_k^{(q)}}_{\text{Collisions}}-\int\overbrace{\Sigma_s^{(q)}(\mathbf{r},\mathbf{v}'\to\mathbf{v})\varphi_k^{(q)}(\mathbf{r},\mathbf{v}')\,\mathrm{d}\mathbf{v}'}^{\text{Scattering}}$$

$$=\frac{\chi^{(q)}(\mathbf{r},v)}{k^{(q)}}\int\underbrace{\nu^{(q)}(v')\Sigma_f^{(q)}(\mathbf{r},v')\varphi_k^{(q)}(\mathbf{r},\mathbf{v}')\,\mathrm{d}\mathbf{v}'}_{\text{Fission}}$$

In compact form: $\boxed{\mathcal{L}^{(q)}\varphi_k^{(q)}(\mathbf{r},\mathbf{v})=\frac{1}{k^{(q)}}\mathcal{F}^{(q)}\varphi_k^{(q)}(\mathbf{r},\mathbf{v})}$

For random geometries: we search the ensemble-averaged eigenpairs

$$\begin{cases}\langle\varphi_k\rangle=\int\mathcal{P}(q)\varphi_k^{(q)}(\mathbf{r},\mathbf{v})dq\\[2ex]\langle k\rangle=\int\mathcal{P}(q)k^{(q)}dq\end{cases}$$

Quenched disorder:

reference solutions

HOMOGENEISATION: « ANNEALED DISORDER »

Instead of solving the exact equation and taking ensemble averages

Introduce **effective transport kernels** $\mathcal{L}'$ and $\mathcal{F}'$

such that $\mathcal{L}'\varphi_k' = \frac{1}{k'}\mathcal{F}'\varphi_k'$ **Annealed disorder**

with the constraints

$$\begin{cases} k' \simeq \langle k \rangle \\ \varphi_k' \simeq \langle \varphi_k \rangle \end{cases}$$

- **Advantages :**
  - solve the eigenvalue problem just once
  - no need to generate random tessellations

- **Drawbacks :**
  - suppressing correlations: no memory of the media already traversed
  - approximate method: need reference solutions for validation

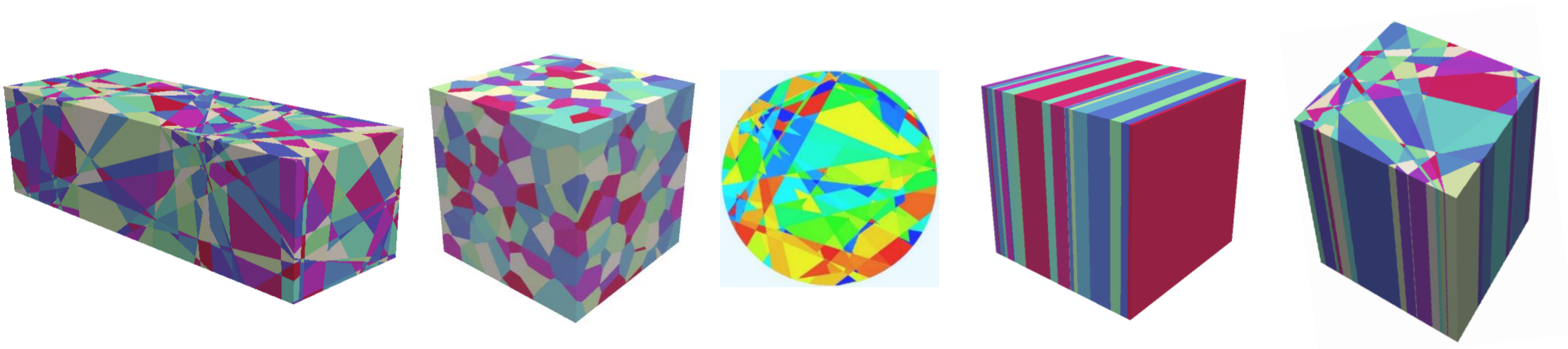
NUMERICAL TOOLS

Eigenvalue problems in stochastic (Poisson) tessellations :

- Analytical results via perturbation theory [Pomraning; Williams]
- Numerical simulations for 1d configurations (slab or rod)

We have developed a **c++ code** to generate random tessellations

- Several **geometrical forms**: d-parallelepipeds, spheres, cylinders, …
- Dimensions: **1d** (rod or slab); **2d** (flat or extruded); **3d**
- Several **mixing statistics**: Poisson (Markov), Voronoi, Box
- **Interface** for the Monte Carlo transport code **TRIPOLI-4**



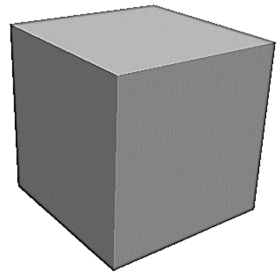
Analysis of Poisson tessellations

d-DIMENSIONAL POISSON TESSELLATIONS

❑ Tessellation of a box of side L

$$\alpha_d = 2\sqrt{\pi}\frac{\Gamma\left(\frac{d+1}{2}\right)}{\Gamma\left(\frac{d}{2}\right)}$$

Initial tessellation : {hyper-cube}



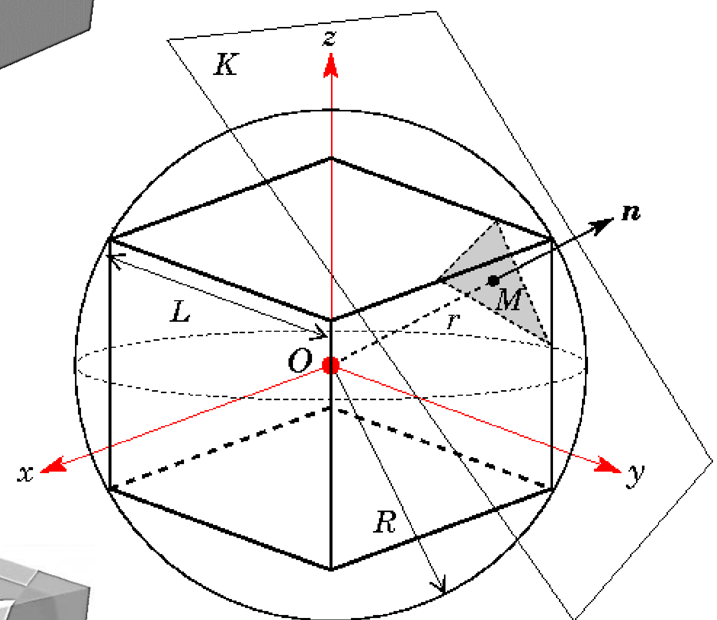
Sampling # of hyper-planes

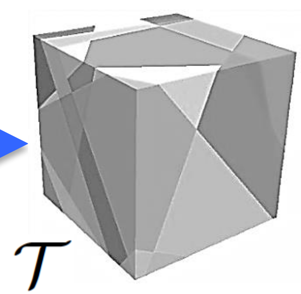
N ~ Poiss(α_d ρ L),

ρ ***tessellation density***

While sampled hyperplanes < N

- Sample a direction **n** ~ H(**n**)
- Intersect the existing polyhedra with the new hyper-plane having *normal vector* **n**
- Update the tessellation



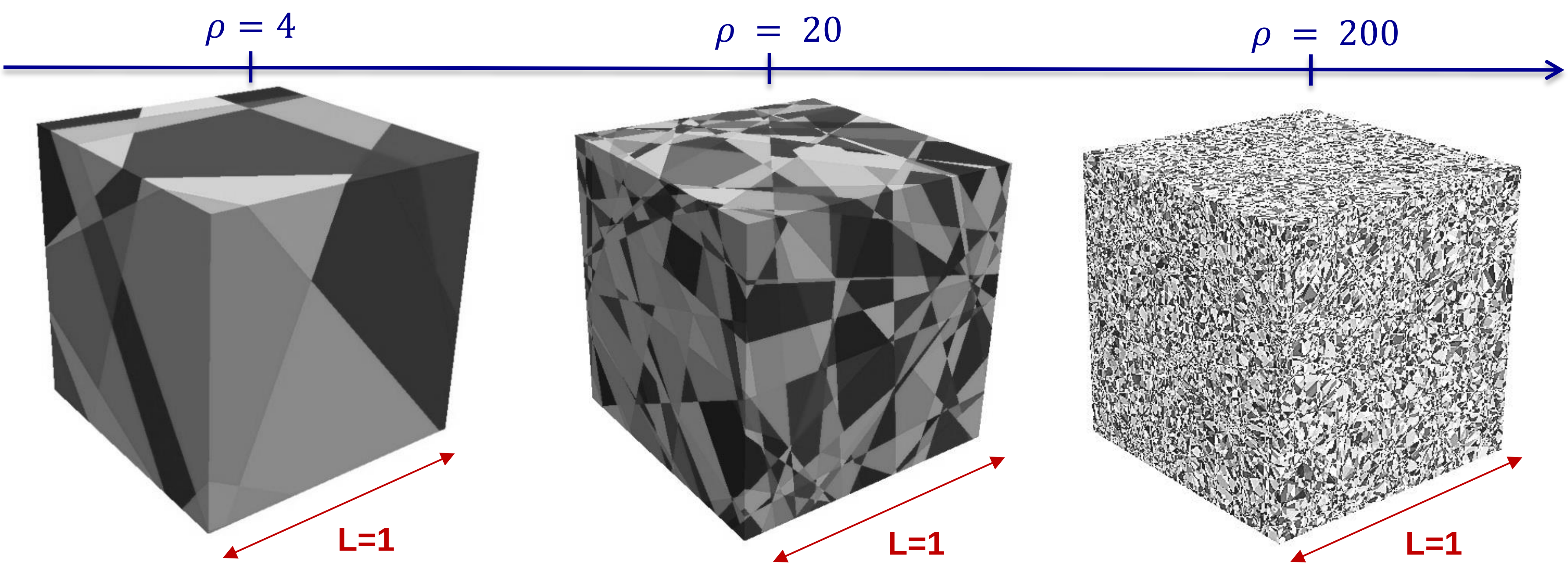


H(**n**) : degree of **anisotropy**

Effects of the tessellation density ρ

- Analysis of the tessellation **density ρ**
  - $1\,/\,\rho\ = \Lambda$ = correlation length = typical disorder size (for infinite tessellations)
  - The number $\mathbf{N_P}$ of **d-polyhedra** increases with increasing ρ **:** $N_p \sim (\rho L)^d$

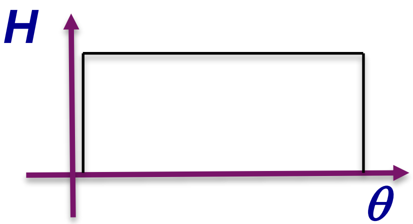
- **3d tessellations :** examples of realizations

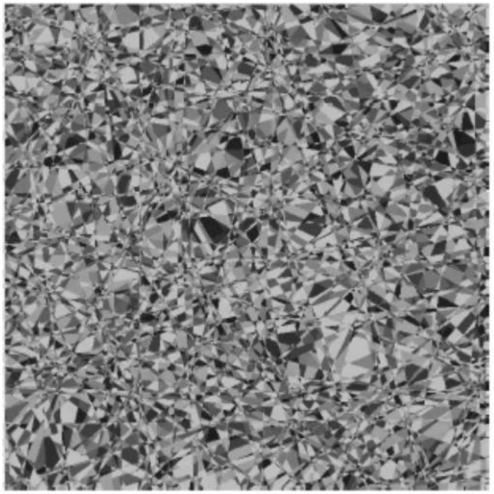


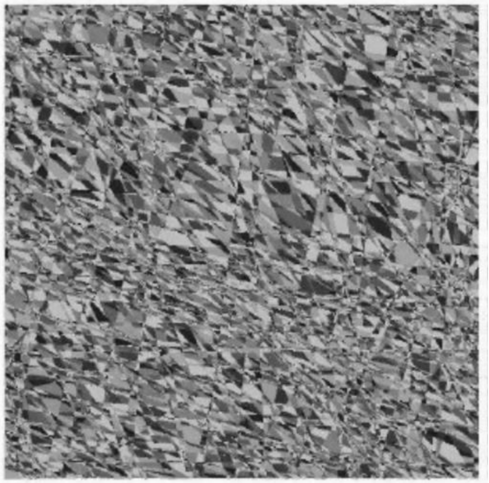
$\sim 10^2$ **polyhedra** $\qquad$ $\sim 5.10^3$ **polyhedra** $\qquad$ ~ 4.106 **polyhedra**

Effects of anisotropy: H(n)

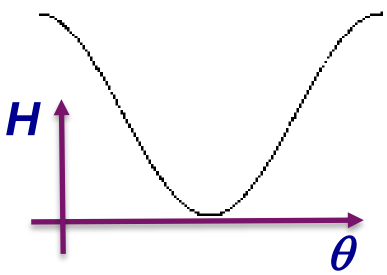
Isotropic



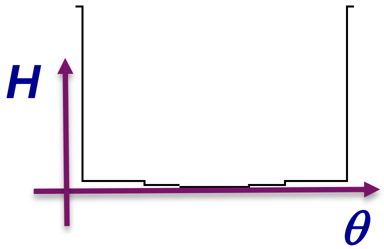


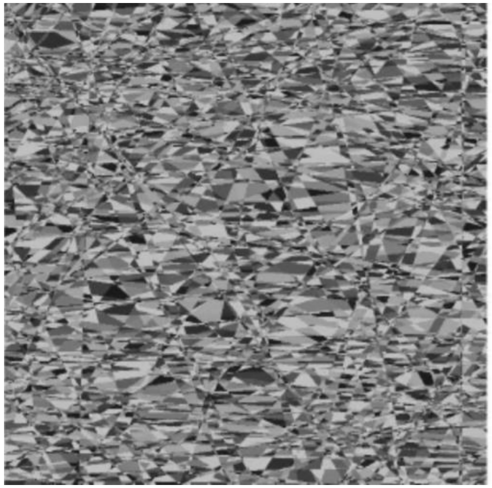


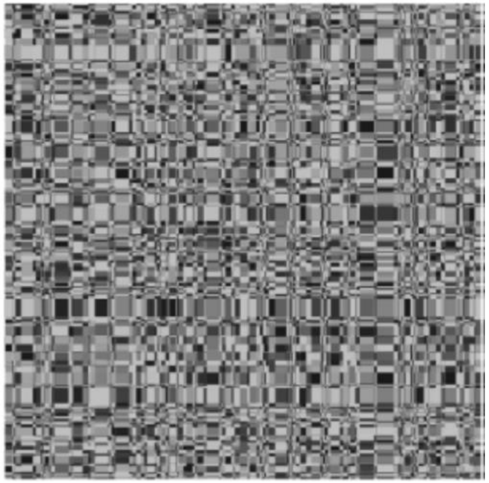
Quadratic



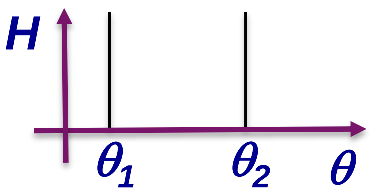
Histogram







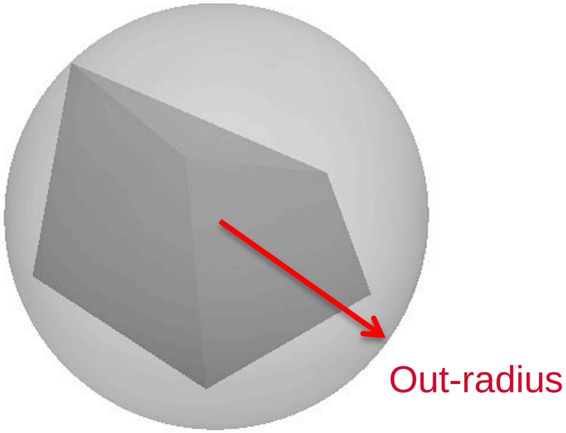
« Box »

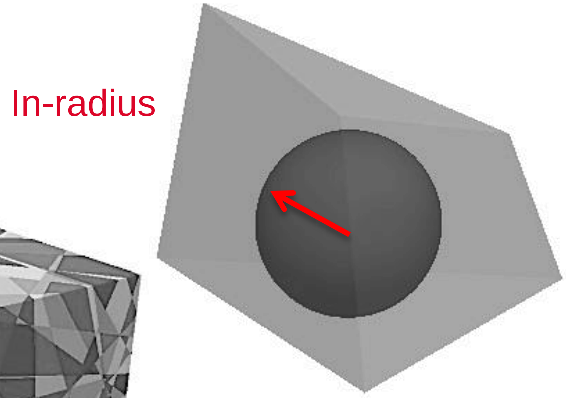


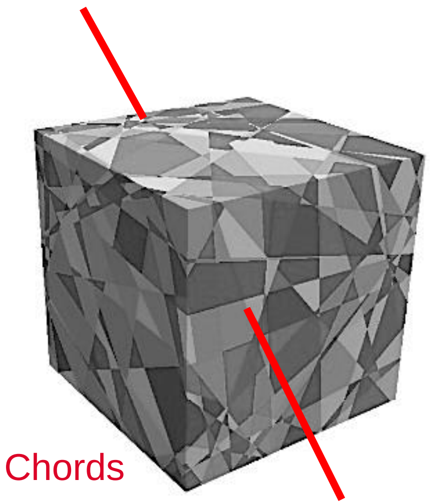
Statistical properties of Poisson tessellations

- Analysis by **Monte Carlo** methods
  - Generate a large ensemble of Poisson tessellations
    - Compute the number of polyhedra
    - Compute the physical observables
  - Estimate **moments** & **distributions**
  - Compare to literature (when available!)

- Relevant observables:
  - Polyhedral features
    - **Volume** and **surface**
    - Number of **faces** (connectivity)
    - ***In-radius / out-radius*** (aspect ratio)
  - Stereology
    - **Chord length** distribution







Polyhedral features [*] of infinite tessellations with law H

[*] Miles (1964); Schneider & Weil (2008)

- Volume: $\langle V_d \rangle = \frac{d!}{\zeta_d} \left(\frac{\alpha_d}{2\rho} \right)^d$ where

$$\zeta_d = \int_{\Omega_d^+} \cdots \int_{\Omega_d^+} \underbrace{[\mathbf{n}_1, \cdots, \mathbf{n}_d]}_{\text{d-space determinant}} dH(\mathbf{n}_1) \cdots dH(\mathbf{n}_d)$$

- Surface: $\langle S_d \rangle = \alpha_d \rho \langle V_d \rangle$

- Number of faces: $\langle f_d \rangle = 2d$

- In-radius: *exponentially* distributed, with $\langle r_{\mathrm{in},d} \rangle = \frac{1}{\alpha_d \rho}$
- Out-radius: unknown

➢ Inequalities: $\langle V_d^m \rangle \geq \langle V_d^m \rangle^{\mathrm{iso}}$

$$\langle S_d \rangle \geq \langle S_d \rangle^{\mathrm{iso}}$$

$$\boxed{\alpha_d = 2\sqrt{\pi}\frac{\Gamma\left(\frac{d+1}{2}\right)}{\Gamma\left(\frac{d}{2}\right)}}$$

Chord lengths [*] for infinite tessellations with law H

[*] Miles (1964); Schneider & Weil (2008)

- A straight line with orientation **v** will encounter a number of hyper-planes having a *Poisson* distribution with density

$$\rho(\mathbf{v}) = \frac{\alpha_d}{2}\rho\int_{\Omega_d^+}|\mathbf{n}\cdot\mathbf{v}|dH(\mathbf{n})$$

$$\boxed{\alpha_d = 2\sqrt{\pi}\frac{\Gamma\left(\frac{d+1}{2}\right)}{\Gamma\left(\frac{d}{2}\right)}}$$

- The line **v** will be cut into chords ℓ *exponentially* distributed with average

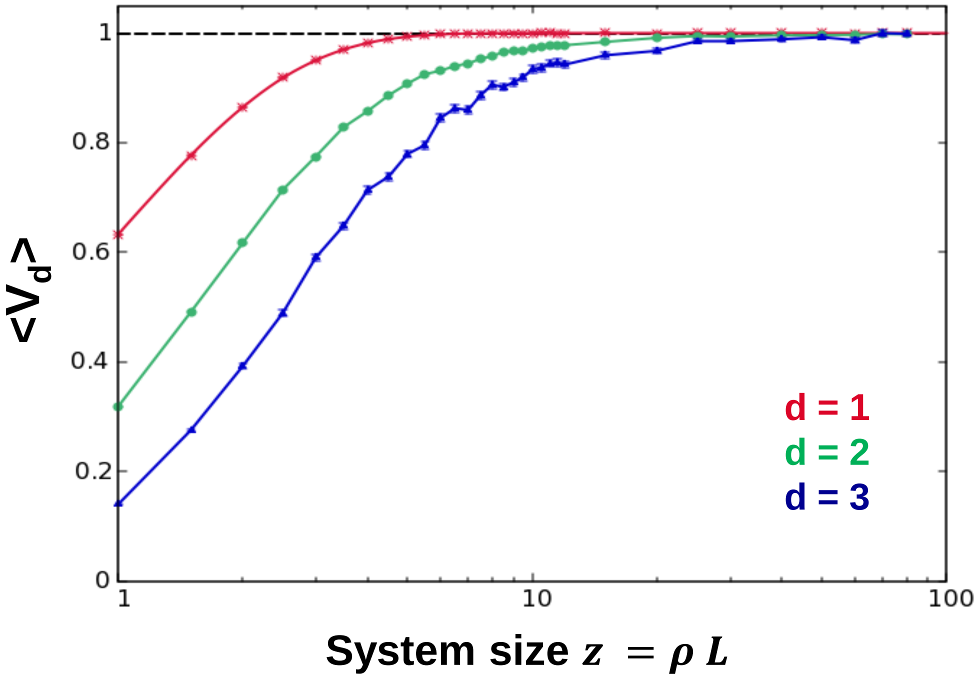
$$\langle\ell\rangle(\mathbf{v}) = 1/\rho(\mathbf{v})$$

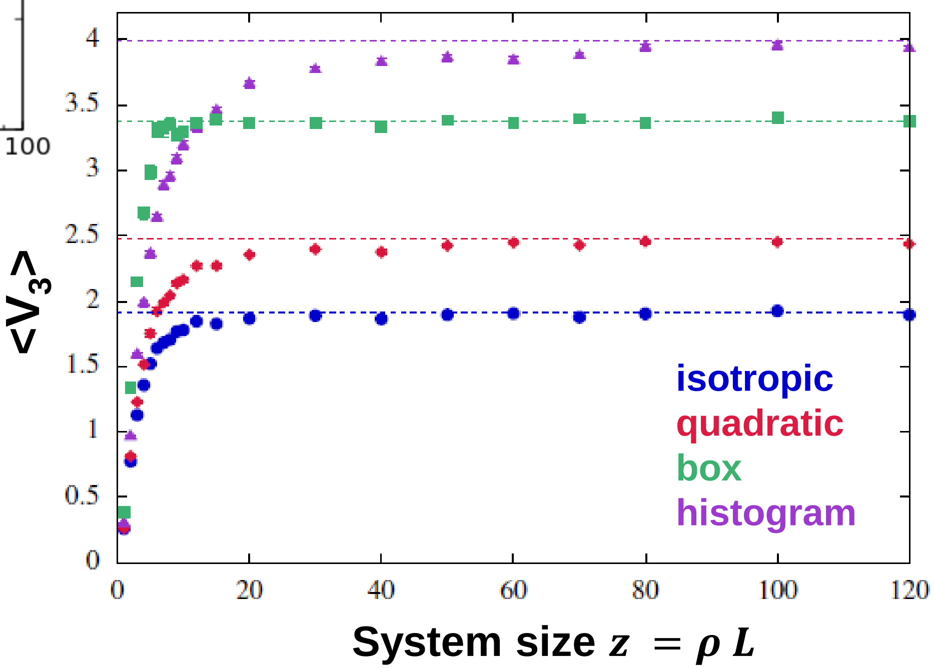
- If the lines **v** are *isotropic and homogeneous*, then

$$\Lambda \equiv \langle\ell\rangle = \alpha_d\frac{\langle V_d\rangle}{\langle S_d\rangle} = \frac{1}{\rho}$$

Correlation length of the tessellation

Finite size effects: polyhedral features



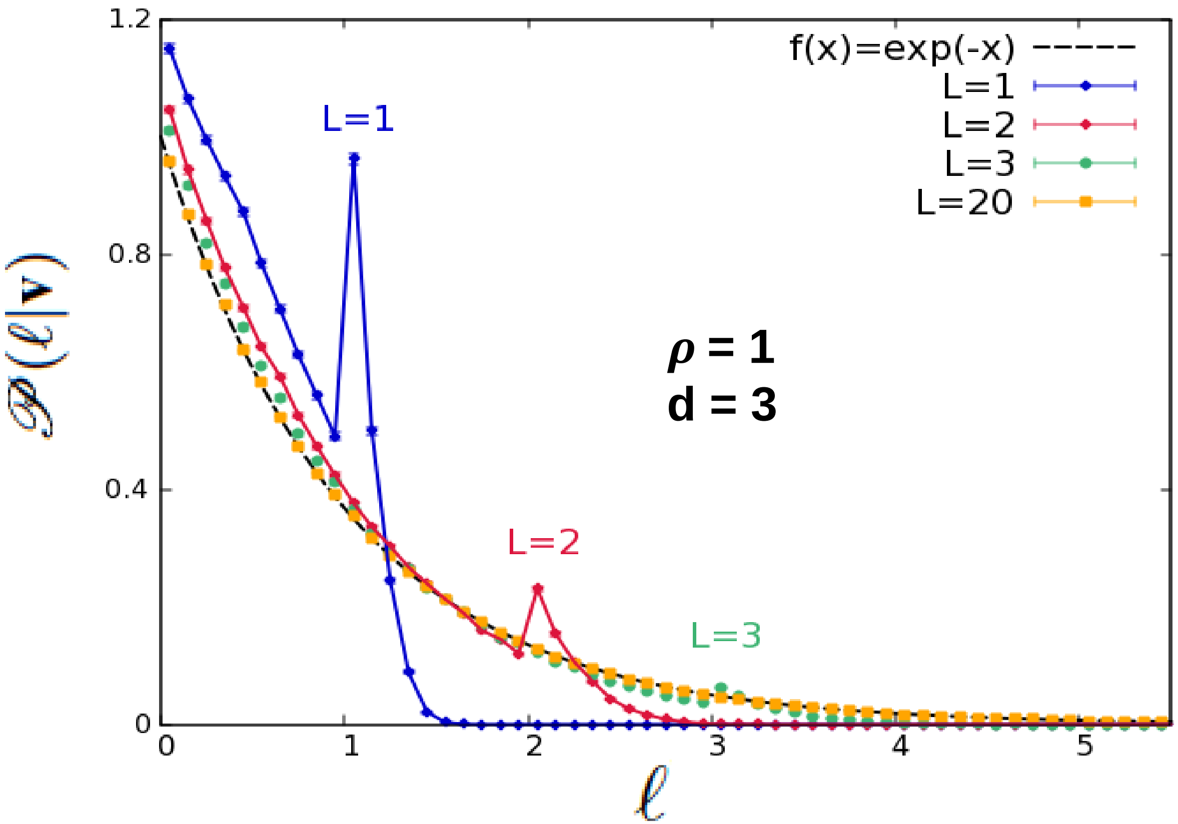


- Monte Carlo results
- Asymptotic values for the moments:

Miles, **Schneider & Weil**

Finite size effects: chord length distribution

Sample lines with direction **v**: $\mathscr{P}(\ell|\mathbf{v}) = \rho(\mathbf{v}) e^{-\ell\rho(\mathbf{v})}$ $\rho(\mathbf{v}) = \frac{\alpha_d}{2}\rho \int_{\Omega_d^+} |\mathbf{n}\cdot\mathbf{v}| dH(\mathbf{n})$



Homogeneous and *isotropic* lines:

H	$\langle \ell \rangle$	Monte Carlo
Isotropic	1	$0.999 \pm 1 \cdot 10^{-3}$
Linear	1	$0.993 \pm 9 \cdot 10^{-4}$
Parabolic	1	$0.993 \pm 1 \cdot 10^{-3}$
Box	1	$0.992 \pm 1 \cdot 10^{-3}$
Histogram	1	$0.993 \pm 1 \cdot 10^{-3}$

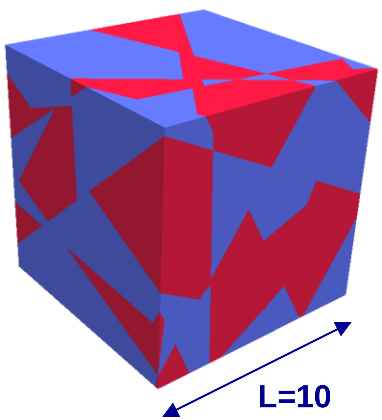
Coloured Poisson geometries and percolation

❑ Attribute **physical properties:**

- Assign each polyhedron a « **label** »
- **Binary** stochastic mixing: **red** with probability **p**, blue with probability 1 - p
- Define a **cluster**: aggregate neighbouring polyhedra sharing the same colour

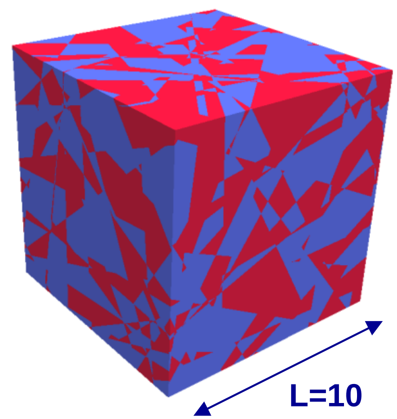
$\rho = 0.6$

$p = 0.50$



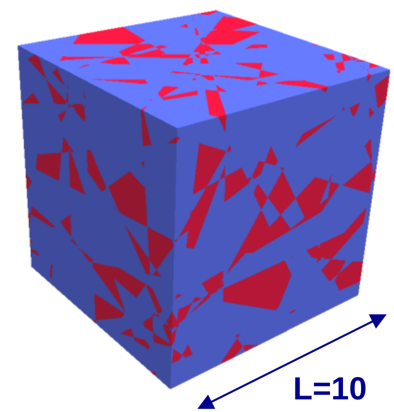
$\rho = 2$

$p = 0.50$



$\rho = 2$

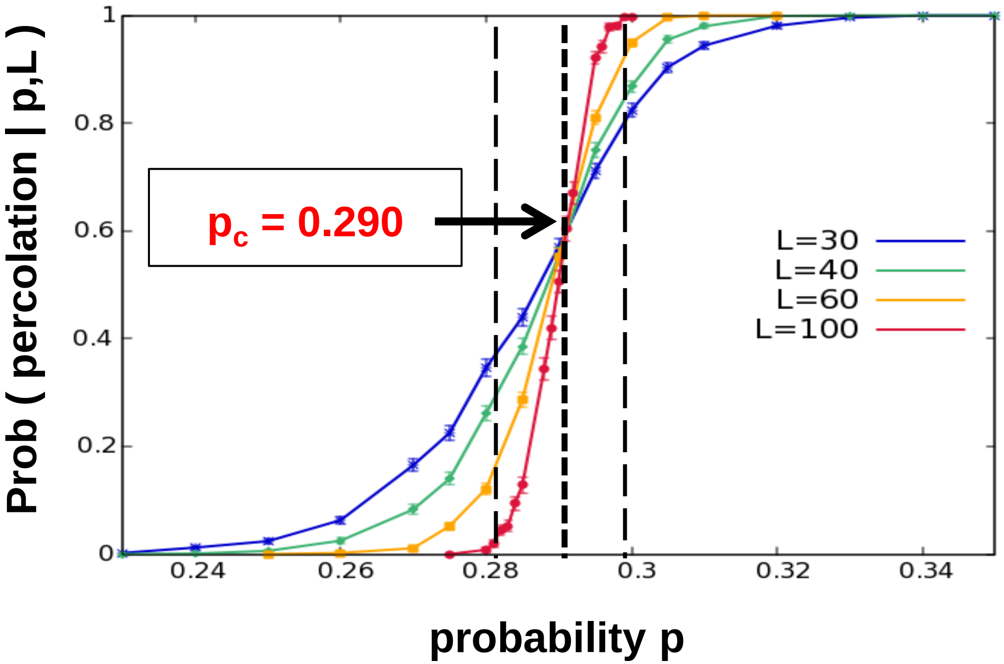
$p = 0.25$



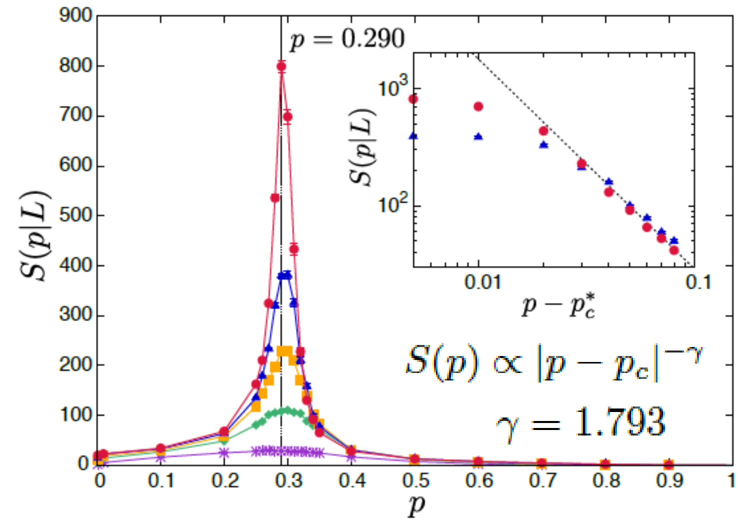
❑ Percolation threshold: p_c

- Value of p beyond which the red cluster spans **a.s.** the entire domain
- Rigorously defined for *infinite* tessellations
- Physical meaning: preferential paths

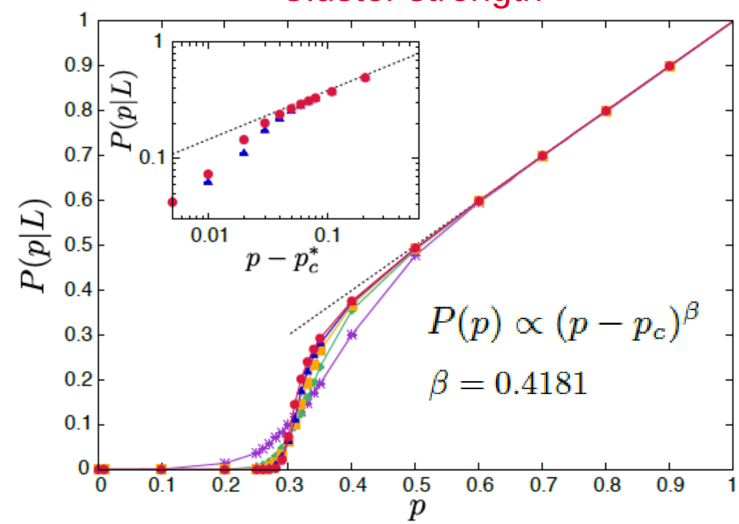
Percolation properties for 3d Poisson geometries



Average cluster size



Cluster strength



3d Poisson : $\mathbf{p_c = 0.290}$ ± 0.007 (Larmier et al., 2016)

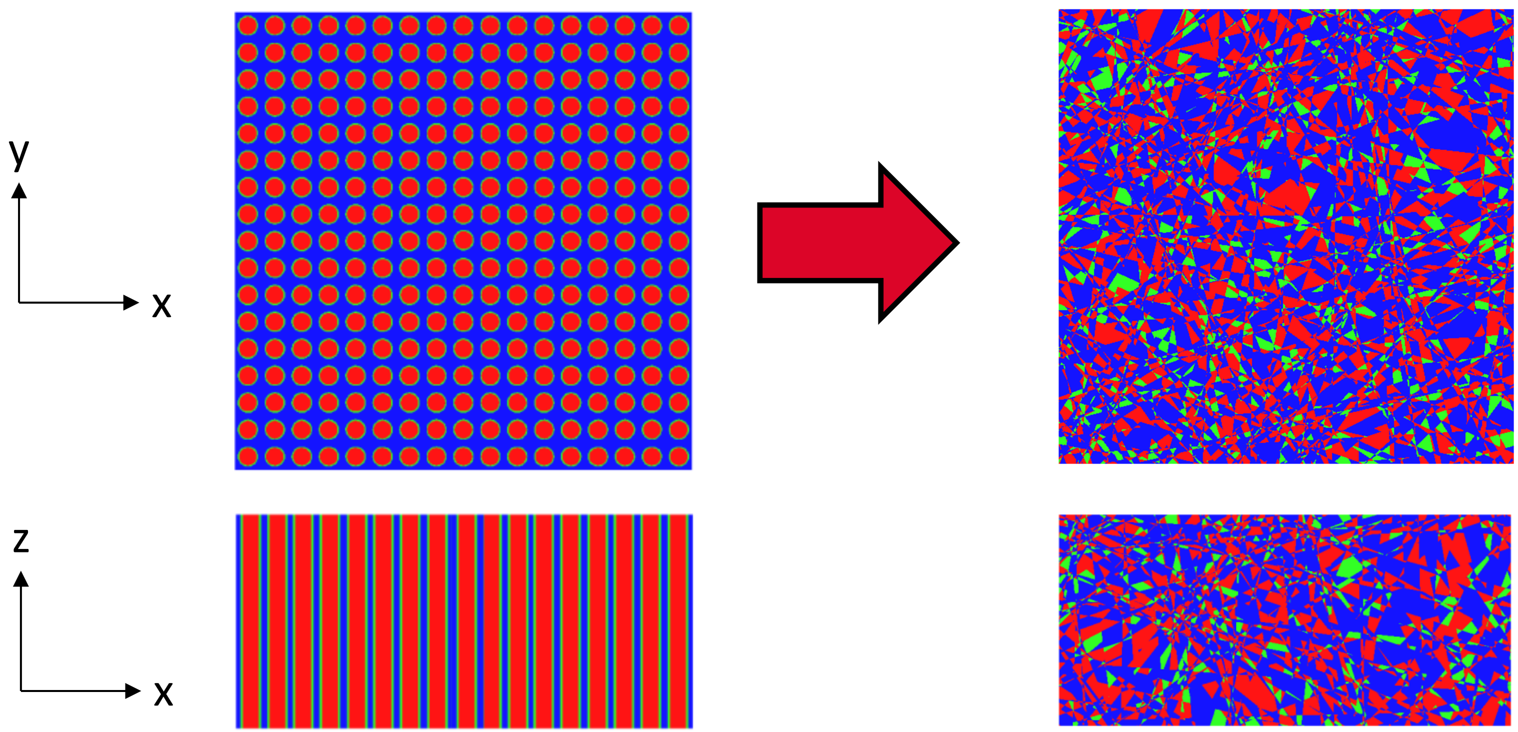
2d Poisson : $p_c = 0.586 \pm 0.001$ (Lepage et al., 2010)

Cubic 3d lattice : $p_c = 0.3126$ (Grassberger, 1992)

Application to an eigenvalue problem

CRITICALITY BENCHMARK: DAMAGED 17x17 FUEL ASSEMBLY

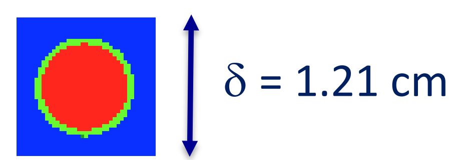
Intact assembly | **Damaged fuel**



Compositions:

(*ternary mixing*, respecting volume fractions **p**)

- **Fuel: UOX/MOX : 35%**
- **Cladding : 10%**
- **Moderator: H_2O : 55%**



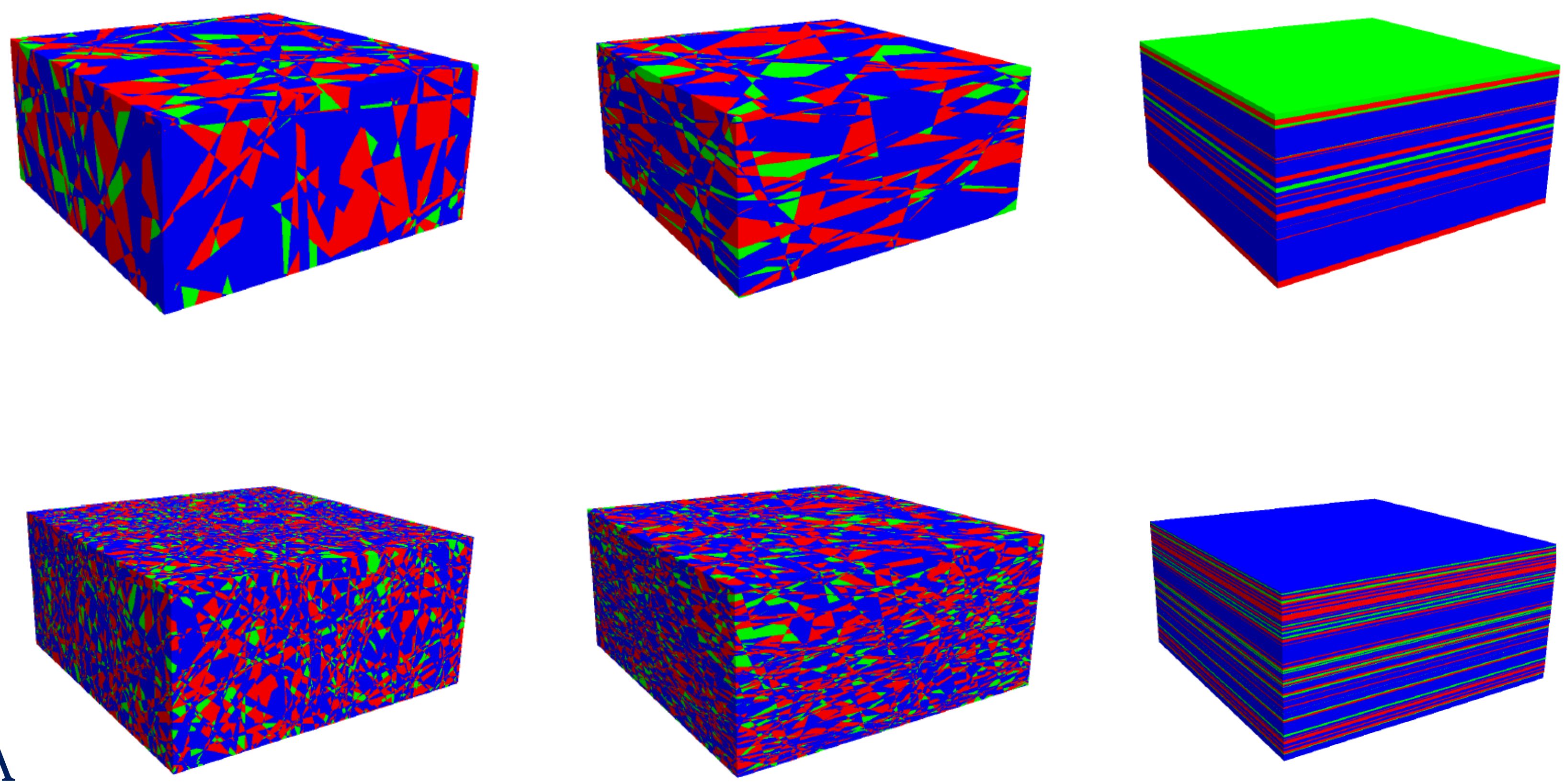
BENCHMARK PARAMETERS

Large Λ

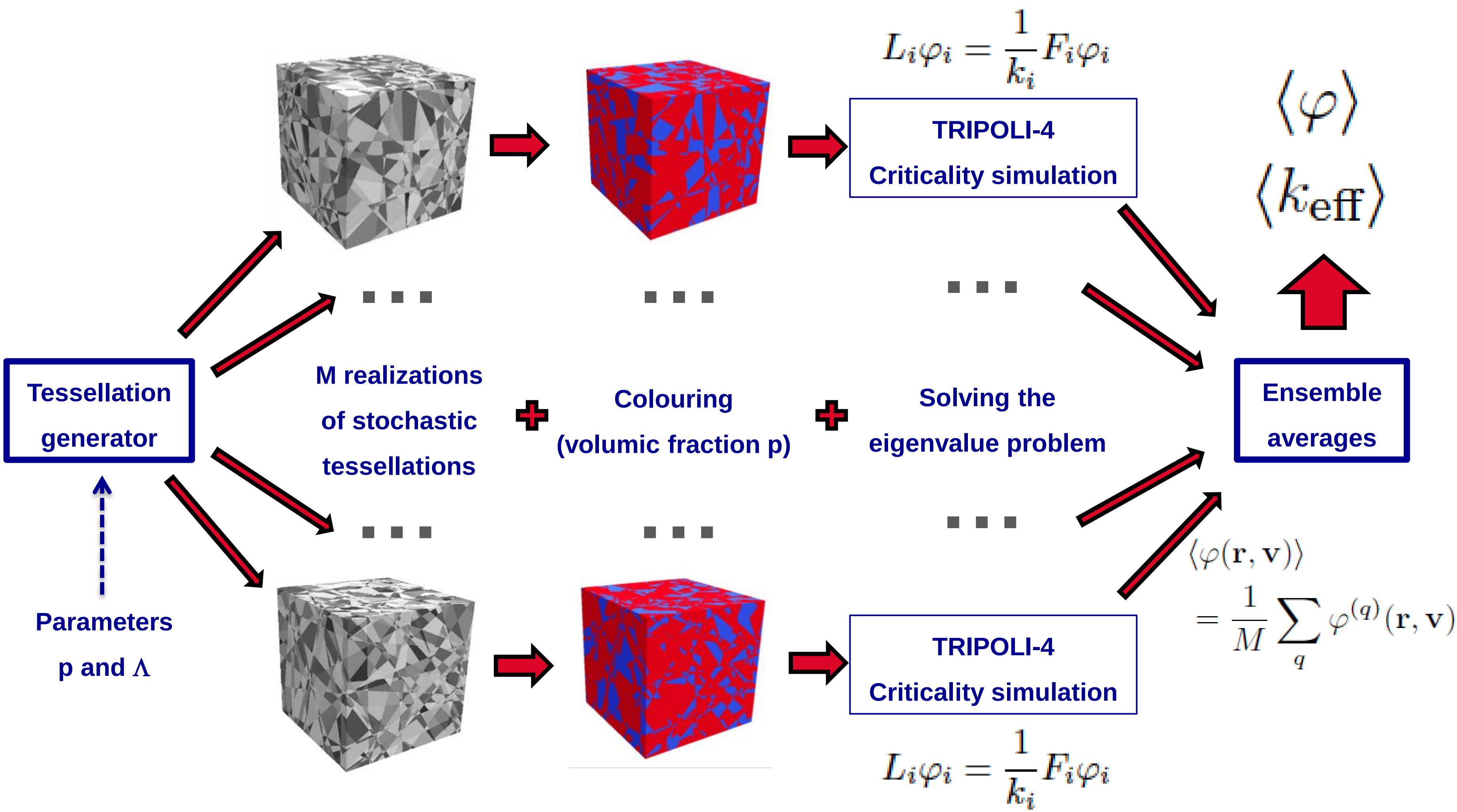
Correlation length Λ

Small Λ

Isotropic tessellations | **Anisotropic tessellations** | **Slab tessellations**

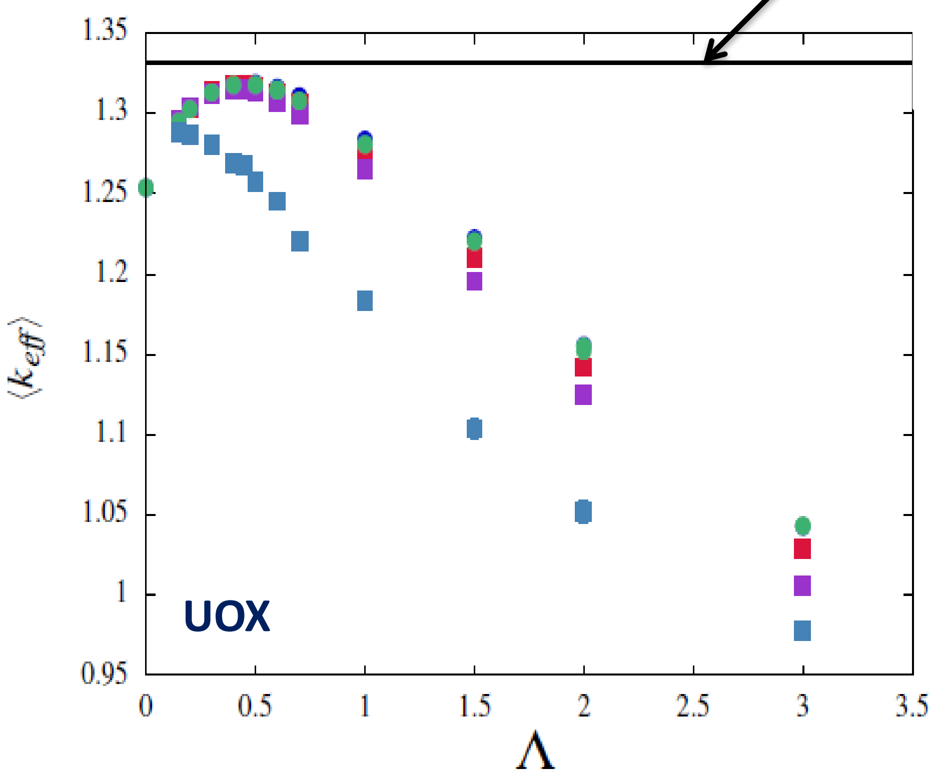


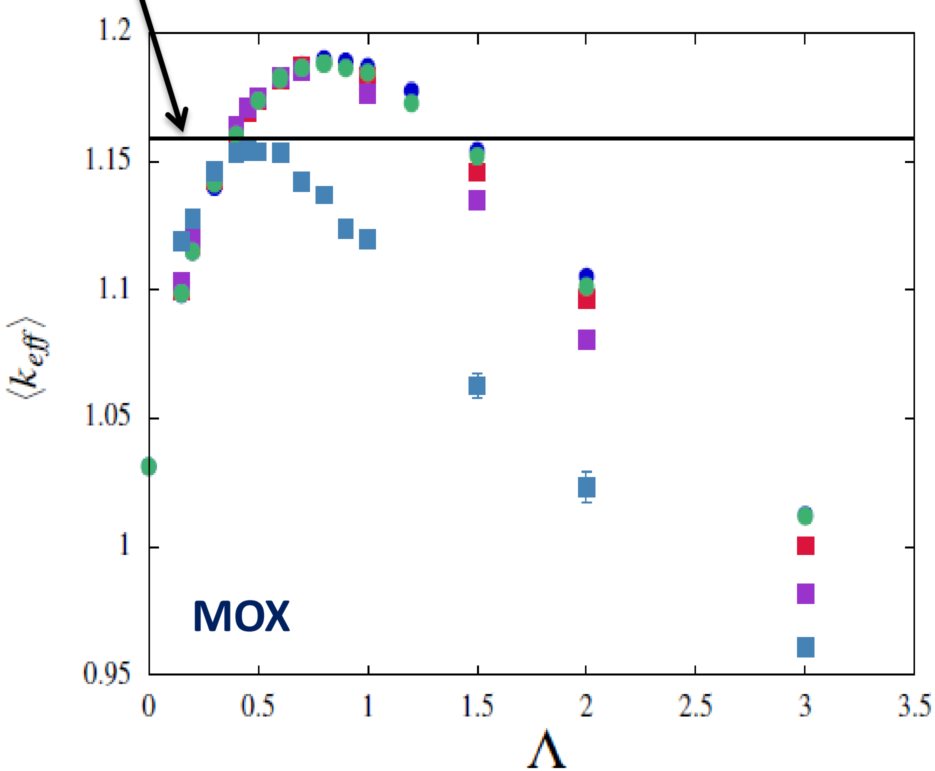
REFERENCE SOLUTIONS BY MONTE CARLO METHODS



AVERAGE EIGENVALUE K_{EFF}

Intact assembly





isotropic
quadratic
box
histogram

slab

Thanks for your attention

References

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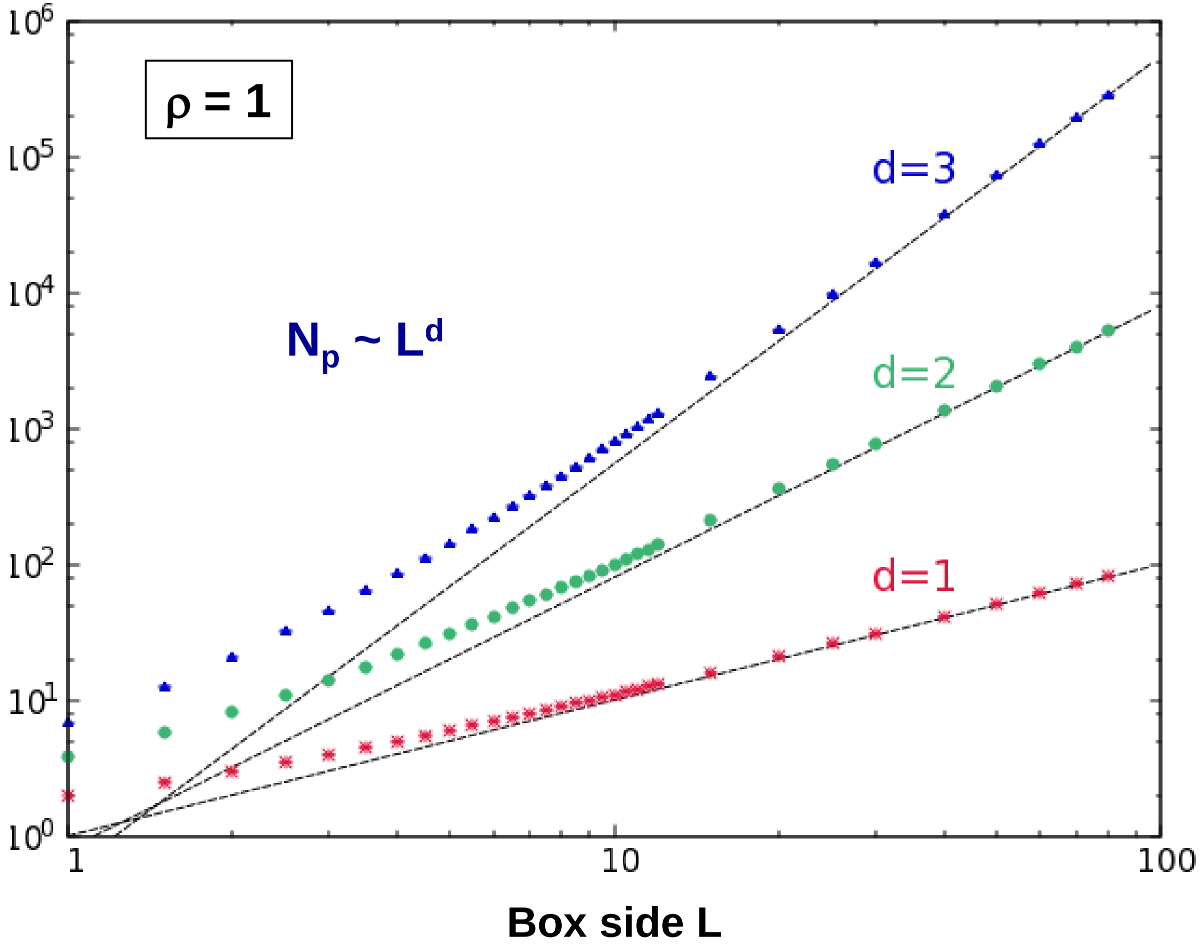
Effects of the tessellation density ρ

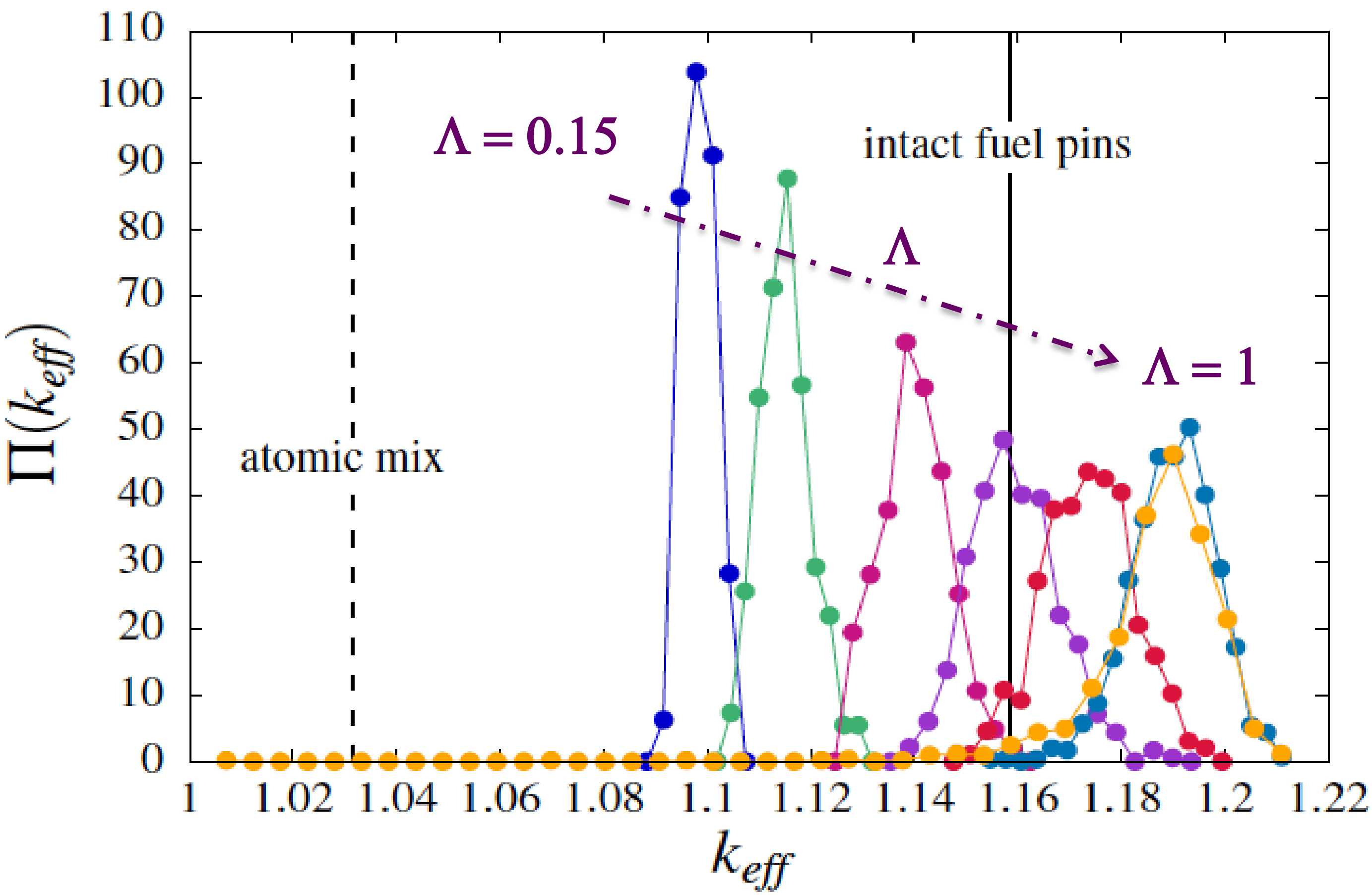
N_P = average number of d-polyhedra



Dimensionless factor $z = \rho L$: system « size »

DISTRIBUTION OF K_{EFF} AS A FUNCTION OF Λ

MOX assembly with **isotropic** Poisson tessellation



DISTRIBUTION OF K_{EFF} AS A FUNCTION OF ANISOTROPY

MOX assembly with **Λ = 1**