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Quantum statistical theory of giant magnetoresistance in magnetic heterogeneous alloys

A. Vedyayev [a,d,*], B. Mevel [a], N. Ryzhanova [b,d], M. Tshiev [d], B. Dieny [a], A. Chamberod [a], F. Brouers [c]

[a] C.E.A. / Département de Recherche Fondamentale sur la Matière Condensée, C.E.N.G., SP2M / MP, 38054 Grenoble cedex 9, France
[b] Laboratoire Louis Néel, C.N.R.S., BP 166X, 38042 Grenoble cedex 9, France
[c] Etude Physique des Matériaux, Institut de Physique, Sart Tilman, Liege 4000, Belgium
[d] Department of Physics, Moscow Lomonosov University, Moscow 119 899, Russian Federation

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Abstract

We present a quantum-statistical theory of giant magnetoresistance in magnetic heterogeneous alloys, consisting of small particles of ferromagnetic metal embedded in a nonmagnetic conducting matrix. The paper focuses on the spin-dependent size-effect on the conductivity of the system, i.e. the influence of the relative orientation of the magnetization in neighboring particles on their conductivity as a function of the size and distance between the particles. It is shown that the conductivity of heterogeneous alloys is not self-averaging when the particle sizes and/or distances are comparable to the mean-free paths. As for current in-plane giant magnetoresistance, the electron mean-free paths are the relevant length scale parameters. When the size-effects due to the bulk and interfacial spin-dependent electron scattering are taken into account, the giant magnetoresistance amplitude exhibits a maximum as a function of the average radius of the particles. The optimum radius value is determined by a balance between the interfacial and bulk scattering.

Keywords: Giant magnetoresistance; Granular alloys

1. Introduction

The phenomenon of giant magnetoresistance (GMR), first discovered in Fe/Cr/Fe [1] multilayers, was later observed in many other multilayers, sandwiches (in particular spin valves) [2] and heterogeneous alloys [3]. These alloys consist of magnetic particles embedded in a non-magnetic conducting matrix, e.g. Co_xAg_{1-x}.

It is now widely believed that the main mechanism giving rise to the GMR in these systems, is the spin-dependent scattering of the conduction electrons by magnetic impurities. The magnetoresistance in heterogeneous alloys is associated with a change in the relative orientation of the magnetization in the neighboring particles. The resistivity is lowest at magnetic saturation and maximum at the coercive field where the magnetic disorder is maximum. Two

* Corresponding author. Fax: +7-095-932-8820; email: vedy@vedy.phys.msu.su.

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contributions to the GMR can be distinguished: one is related to the spin-dependent electron scattering on the interfacial impurities, the other is due to the bulk scattering.

The non-local conductivity is inhomogeneous on the scale of the mean-free paths in the various materials [4]. The GMR is due to the classical spin-dependent size-effect in electron transport. In magnetic multilayers, it has been pointed out [4] that the two geometries of current in plane (CIP) and current perpendicular to the plane (CPP) give completely different size-effects. In the former case, the size-effects are important. The mean-free paths are the relevant length scales. In contrast, for the CPP geometry, the resistivity of the system may be calculated by assuming that the layers just form a series of resistances, in the limit where the thickness of the various layers is much smaller than the electron mean-free paths. This limit is most often relevant in experiments since the spin-diffusion length in commonly used metals is of the order of several hundred Å [5]. In other words, the scattering rate of the conduction electrons is the averaged sum of the scattering rates in the various parts of the system (ferromagnetic and paramagnetic layers and at the interfaces). In this case, the resistance of the system does not depend on the ratios of the thickness of the layers divided by the mean-free paths. Levy et al. [6,7] suggested that one may apply the same formalism for heterogeneous alloys as for the CPP geometry in multilayers. Following this assumption, the inverse mean-free path in the system is proportional to the weighted sum of the scattering rates for all scattering centers whether in the bulk of the particles, in the matrix or at the interfaces. Proceeding further, they assumed that the ratio of the spin-dependent scattering potential to the spin-independent potential is larger for surface defects than for bulk defects. Since the number of surface atoms increases with decreasing mean radius of the granules, the GMR was found to be approximately proportional to the inverse of the radius. To obtain agreement with most experiments in which a maximum of GMR is observed as a function of the annealing temperature and therefore as a function of the particle radius [3], the authors suggested that in fields up to 20 kOe, the magnetization of the small magnetic precipitates is not saturated. The smallest particles, therefore, do not contribute to the GMR. This provides a possible explanation for the initial increase in GMR amplitude as a function of annealing temperature.

However, it remains unclear why the classical size-effects, which are important for the CIP geometry in multilayers, would not play any role in heterogeneous alloys. Indeed, the transport mechanism in these alloys seems intuitively intermediate between the CIP (current flowing parallel to the magnetic inhomogeneities) and the CPP (current flowing perpendicular to the magnetic inhomogeneities) geometries in multilayers.

In fact, if the sizes and/or distances between the magnetic particles are much smaller than all mean-free paths of spin $\uparrow$ and spin $\downarrow$ electrons either in the particles or in the matrix, then the global conductivity of the system may be considered as the conductivity of an effective medium characterized by a single mean-free path for each direction of electron spin. This situation corresponds to the self-averaging limit [4]. In contrast, if these distances are comparable to or larger than the mean-free paths, the current lines will avoid the regions with higher resistivity. The situation will then be closer to the CIP geometry in multilayers. So far, this case has not been investigated from a theoretical point of view. Consequently, to deal with this particular situation, we investigate the GMR of heterogeneous alloys in the framework of the Kubo formalism by calculating the non-local two points conductivity $\sigma(r, r')$ which depends on the position vectors $\boldsymbol{r}$ and $\boldsymbol{r}'$ either in the particles or in the matrix, and on the values of the mean-free paths of spin $\uparrow$ and spin $\downarrow$ electrons in the various parts of the system.

Initially, we assume that the electron mean-free path in the paramagnetic matrix does not depend on the spin direction or on the radius of the particles. Only the bulk scattering in the ferromagnetic particles is considered as spin-dependent. In this case, the magnetoresistance is due to the reciprocal influence of neighboring particles on the conductivity within each particle. This influence depends on the relative orientation of the magnetization in a particle with respect to the magnetization in neighboring particles (parallel at saturation or random in zero field). In this calculation, we do not assume that the inverse mean-free paths are self-averaging quantities for any distance $|r - r'|$ as was done in Refs. [6,7]. In con-

trast, we calculate the non-local conductivity exactly for distances $|r-r'|$ smaller than the mean distance between neighboring particles and use average mean-free paths for spin ↑ and spin ↓ electrons for larger distances.

Next, we use the effective medium approach to calculate the global conductivity which is measured experimentally. It has been proven that this approach is appropriate for the calculation of the conductivity of systems composed of a random mixture of two components with different conductivities [8].

We then introduce electron mean-free paths in the matrix, which depend on the size of the granules due to surface scattering. This scattering may be different for spin ↑ and spin ↓ directions for ferromagnetic alignment (interfacial spin-dependent scattering). Thus, we can calculate the GMR when both mechanisms are involved: spin-dependent size-effect within the non-local approach of the electron transport theory and spin-dependent surface scattering as a self-averaging contribution to the resistivity as in CPP transport. We show that this non-local theory predicts that the dependence of the GMR amplitude on the radius of the magnetic particles exhibits a maximum for an optimum value of the radius, which is determined by a balance between bulk and surface electron scattering mechanisms.

2. Model calculation

We describe the heterogeneous alloy as a matrix of non-maqnetic metal containing randomly distributed spherical ferromagnetic particles of radius r_0. The magnetic particles are assumed to be single domain. They occupy a volume fraction x. For simplicity, we assume that the magnetization of the particles can be aligned either parallel or antiparallel to the applied field. For a fraction $1+L$ of the particles, the magnetization is parallel to the field. For the other $(1-L)$ particles, the magnetization is antiparallel. The net magnetization is a linear function of L. The probability of antiparallel alignment between neighboring particles is then quadratic in L. Therefore, the resistance is expected to vary as M^2 for this model in agreement with the experimental observations [3]. In this first study of size-effect on the transport properties in heterogeneous alloys, we did not consider non-collinear configurations of the magnetization of the granules. Taking these configurations into account should not change the main physical ideas outlined in this paper concerning the comparison of the mean-free paths and size of the particles.

As a first approximation, we adopt a free electron model. The Fermi momenta $\hbar k_F$ of the electrons are assumed equal for spin ↑ and spin ↓ electrons within the matrix and in the particles. However, their mean-free paths are different. In this case, the non-local conductivity of the system may be calculated from the Kubo formula in the form suggested in Ref. [9]:

$$\sigma(r,r') = -\frac{4e^2}{\pi\hbar}\left(\frac{\hbar^2}{2m}\right)\left[G^{\mathrm{ret}}(r,r') - G^{\mathrm{adv}}(r,r')\right] \times \overleftrightarrow{\nabla}r\overleftrightarrow{\nabla}r'\left[G^{\mathrm{ret}}(r,r') - G^{\mathrm{adv}}(r,r')\right], \tag{1}$$

where $\overleftrightarrow{\nabla}r = \frac{1}{2}(\overrightarrow{\nabla}r - \overleftarrow{\nabla}r)$ is the antisymmetric gradient operator and G^{ret} and G^{adv} are the retarded and advanced Green functions of the system. These Green functions are the solutions of the equation:

$$\left[\nabla_r^2 + k^2(r)\right]G(r,r') = \frac{2m}{\hbar^2}\delta(r-r'), \tag{2}$$

where $k^2(r) = k_F^2 + \mathrm{i}\Delta(r)$. $\hbar k_F$ is the electron Fermi momentum. $\Delta(r) = (2k_F/l(r))$, where $l(r)$ is the electron local mean-free path. If $\Delta(r) \ll k_F^2$, i.e. $k_F l(r) \gg 1$, which is fulfilled for all common metals, the solution of Eq. (2) can be found within the WKB approximation:

$$G(r,r') = \frac{1}{4\pi|r-r'|}\mathrm{e}^{\mathrm{i}k_F|r-r'|}\mathrm{e}^{-\xi(r,r')|r-r'|}, \tag{3}$$

with

$$\xi(r,r') = \frac{1}{4\pi|r-r'|k_F}\int_{\Gamma(r,r')}\mathrm{d}s''\Delta(r'').$$

In this expression, the integral is taken along the straight path $\Gamma(r,r')$ starting at point r' and ending at point r. Eqs. (1)–(3) are quite general. They have already been used in Ref. [7]. However, from this

point, our calculations differ from those of Ref. [7]. Indeed, in the latter, $\xi(r,r')$ was approximated according to $\xi(r,r') = \overline{(1/l(r))}$ for any value of $|r-r'|$. In this expression, $\overline{(\,)}$ denotes the average over the total volume of the system, so that $\overline{(1/l(r))}$ does not depend on r. In contrast to Ref. [7], we take into account the spatial variation of $\xi(r,r')$. For that, we distinguish the asymptotic behavior of the Green's function at small and large distances:

$$G(r,r') = \frac{e^{ik_F|r-r'|}e^{-|r-r'|/l_{1(3)}}}{|r-r'|}, \tag{4}$$

where r and r' are taken inside the same particle, $l_{1(3)}$ is the electron mean-free path for spin $\uparrow$ ($\downarrow$) electrons;

$$G(r,r') = \frac{e^{ik_F|r-r'|}}{|r-r'|}e^{-r\cos(\theta)/l_{1(3)}}e^{-r'/l_2} \times e^{[(1/l_2)-(1/l_{1(3)})]r_0}, \tag{5}$$

where r is taken inside the particle, r' in the matrix, and $|r-r'| < R + (r_0/2)$. Θ represents the angle between r and r' and R is the average distance between the centers of nearest neighbor particles. l_2 is the electron mean-free path in the matrix;

$$G(r,r') = \frac{e^{ik_F|r-r'|}}{|r-r'|}e^{-r\cos(\theta)/l_{1(3)}} \times e^{-r'/l_{1(3)}}e^{[(1/l_{1(3)})-(1/l_2)]R} \times e^{[(2/l_2)-(1/l_{1(3)})-(1/l_{3(1)})]r_0}, \tag{6}$$

where r is inside the particle and r' inside another nearest neighbor particle at distance R; and

$$G(r,r') = \frac{e^{ik_F|r-r'|}}{|r-r'|}e^{i|r-r'|\bar{l}^{-1}_{\uparrow(\downarrow)}}, \tag{7}$$

for

$$|r-r'| > R + \frac{r_0}{2},$$

where $\bar{l}^{-1}_{\uparrow(\downarrow)} = (x/2)(1+L)l^{-1}_{1(3)} + (x/2)(1-L)l^{-1}_{3(1)} + (1-x)l_2^{-1}$ represents the average inverse electron mean-free path for an up/down spin electron.

Now, by substituting Eqs. (4)–(7) into Eq. (1), integrating over the coordinate r' and averaging over coordinate r, we obtain expressions for the local conductivities $\sigma_{gr\uparrow}^{\uparrow(\downarrow)}$, $\sigma_{gr\downarrow}^{\uparrow(\downarrow)}$ and $\sigma_M^{\uparrow(\downarrow)}$ within the magnetic particles with magnetization parallel and antiparallel to the magnetic field and for the matrix for up(down) spin electrons:

$$\begin{aligned}
\sigma_{gr\uparrow}^{\uparrow} = \frac{e^2k_F^2}{6\pi^2\hbar}\Bigg\{ & l_1 - \frac{3}{4}\left(\frac{l_1}{r_0}\right)^2 F\left(2\frac{r_0}{l_1}\right)e^{-2r_0/l_1} \\
& \times\Bigg[l_1 - l_2(1-e^{-2(R/l_2)}) \\
& -\bar{l}^{\uparrow}\Bigg(e^{-2\{[(R-2r_0)/(l_2^{\uparrow})]-4(r_0/l_1)\}}\frac{x}{2}(1+L) \\
& + e^{-2\{[(R-2r_0)/(l_2^{\uparrow})]-4(r_0/l_3)\}}\frac{x}{2}(1-L) \\
& + (1-x)e^{-2R/l_2^{\uparrow}}\Bigg)\Bigg] + \frac{3}{8}\frac{l_1^2}{r_0R^2} \\
& \times F\left(2\frac{r_0}{l_1}\right)e^{-2R/l_2^{\uparrow}}e^{2r_0[(1/l_2^{\uparrow})-(1/l_1)]} \\
& \times\Bigg[\left(l_1^2e^{2r_0[(1/l_2^{\uparrow})-(1/l_1)]}F\left(2\frac{r_0}{l_1}\right)\right. \\
& \left. - l_2^{\uparrow 2}F\left(2\frac{r_0}{l_2^{\uparrow}}\right)\right)N^{\uparrow} \\
& + \left(l_3^2e^{2r_0[(1/l_2^{\uparrow})-(1/l_3)]}F\left(2\frac{r_0}{l_3}\right)\right. \\
& \left. - l_2^{\downarrow 2}F\left(2\frac{r_0}{l_2^{\downarrow}}\right)\right)N^{\downarrow}\Bigg]\Bigg\},
\end{aligned} \tag{8}$$

in which $F(x) = \mathrm{ch}(x) - (\mathrm{sh}(x)/x)$. The upper $\uparrow$ in $\sigma_{gr\uparrow}^{\uparrow}$ denotes the direction of the electron spin while the lower $\uparrow$ represents the direction of the magnetization in the particle. Furthermore, $N^{\uparrow}$ and $N^{\downarrow}$ designate the average number of nearest neighbor magnetic particles with up and down magnetizations respectively. They are defined by $N^{\uparrow} = x(1+L)[3(4\pi/3x)^{2/3}+1]$ and $N^{\downarrow} = x(1-L)[3(4\pi/3x)^{2/3}+1]$. The average distance R be-

tween the centers of the particles is given by $R = (4\pi/3x)^{1/3} r_0$.

$$\sigma_M^{\uparrow} = \frac{e^2 k_F^2}{6\pi^2 \hbar} \left\{ l_2 - \frac{3}{4} \frac{l_2^2}{[(R/2) - r_0]^2} \right.$$
$$\times F\left(2\frac{[(R/2) - r_0]}{l_2}\right) e^{-2[((R/2) - r_0)/l_2]}$$
$$\times \left[l_2 - \bar{l}^{\uparrow} \left((1-x) e^{-4(r_0/l_2)} + \frac{x}{2}(1+L) \right.\right.$$
$$\left.\left. \times e^{-4(r_0/l_1)} + \frac{x}{2}(1-L) e^{-4(r_0/l_3)} \right) \right]$$
$$+ \frac{3}{2} \frac{r_0 l_2^2}{R^2 [(R/2) - r_0]^2} e^{-R/l_2} F$$
$$\times \left(2\frac{[(R/2) - r_0]}{l_2} \right) \left[\left(e^{-2r_0[(1/l_1) - (1/l_2)]} l_1^2 F \right.\right.$$
$$\left. \times \left(2\frac{r_0}{l_1} \right) - l_2^2 F\left(2\frac{r_0}{l_2} \right) \right) N^{\uparrow *}$$
$$+ \left(e^{-2r_0[(1/l_3) - (1/l_2)]} l_3^2 F\left(2\frac{r_0}{l_3} \right) \right.$$
$$\left.\left.\left. - l_2^2 F\left(2\frac{r_0}{l_2} \right) \right) N^{\downarrow *} \right] \right\}, \tag{9}$$

in which $N^{\uparrow *} = x(1+L)[(3/4)(4\pi/3x)^{2/3} + 1]$ and $N^{\downarrow *} = x(1-L)[(3/4)(4\pi/3x)^{2/3} + 1]$.

The expressions for all other local conductivities $\sigma_{gr\uparrow}^{\uparrow}$, $\sigma_{gr\uparrow}^{\downarrow}$, $\sigma_{gr\downarrow}^{\downarrow}$ and $\sigma_M^{\downarrow}$ may be written in forms similar to Eqs. (8) and (9) by interchanging $l_1 \leftrightarrow l_3$ and $\bar{l}^{\uparrow} \leftrightarrow \bar{l}^{\downarrow}$. In Eq. (8) the first term is just the bulk conductivity of the spin ↑ electrons. The second term is responsible for the classical size-effects which are due to the difference between the conductivity of the magnetic granules and that of both the non-magnetic matrix and the effective medium (or effective matrix), which is characterized by the average of the inverse mean-free path $\bar{l}^{-1}$. The third term is due to the influence on the conductivity of the considered particle, of the presence, at distance R, of other particles with up and down magnetizations. At this point, it is important to note that, in the limit $2r_0/l_i \to 0$, all local conductivities for spin ↑(↓) electrons have the limiting value $(e^2 k_F^2/6\pi^2\hbar)(1/\bar{l}^{-1}_{\uparrow(\downarrow)})$. It is only in this limit of small particle size as compared to the mean-free paths that we recover the results of Levy et al. [6,7]. Our results have a clear physical meaning: for mean-free paths much larger than the scale of the inhomogeneity in the density of scattering centers, the system can be approximated by a simple average medium characterized by a scattering rate equal to the average of the local scattering rates, i.e. a mean-free path equal to the inverse of the average of inverse local mean-free paths. In contrast, if the mean-free paths are smaller than the characteristic sizes of these inhomogeneities, namely r_0 and R, all local conductivities recover their initial bulk values and all terms in Eqs. (8) and (9) containing the factor L, which characterizes the degree of polarization of the magnetization of the magnetic particles, decrease proportionally to $(l_i/r_0)^2 \exp(-R/l_2)$ ($i = 1$, 2 or 3).

In this latter case, we have to determine the global conductivity of the system, the local conductivities being defined by Eqs. (8) and (9). To calculate it, we use the effective medium approach as developed in Ref. [8]. Following this approach, the effective macroscopic conductivity $\sigma_{ef}^{\uparrow(\downarrow)}$ for electrons with spin ↑(↓) is the solution of the equations:

$$\frac{x}{2}(1+L)\frac{\sigma_{gr\uparrow}^{\uparrow(\downarrow)} - \sigma_{ef}^{\uparrow(\downarrow)}}{\sigma_{gr\uparrow}^{\uparrow(\downarrow)} + 2\sigma_{ef}^{\uparrow(\downarrow)}}$$
$$+ \frac{x}{2}(1-L)\frac{\sigma_{gr\downarrow}^{\uparrow(\downarrow)} - \sigma_{ef}^{\uparrow(\downarrow)}}{\sigma_{gr\downarrow}^{\uparrow(\downarrow)} + 2\sigma_{ef}^{\uparrow(\downarrow)}}$$
$$+ (1-x)\frac{\sigma_M^{\uparrow(\downarrow)} - \sigma_{ef}^{\uparrow(\downarrow)}}{\sigma_M^{\uparrow(\downarrow)} + 2\sigma_{ef}^{\uparrow(\downarrow)}} = 0. \tag{10}$$

By solving these two self-consistent equations, we can find the relative magnetoconductance defined by $\Delta\sigma/\sigma(L=1) = [\sigma(L=1) - \sigma(L=0)]/\sigma(L=1)$, where $\sigma = \sigma_{ef}^{\uparrow} + \sigma_{ef}^{\downarrow}$. We have done it numerically for a set of parameters as indicated in the caption of Fig. 1, which shows the dependence of $\Delta\sigma/\sigma(L=1)$ versus the radius r_0 of the magnetic particles.

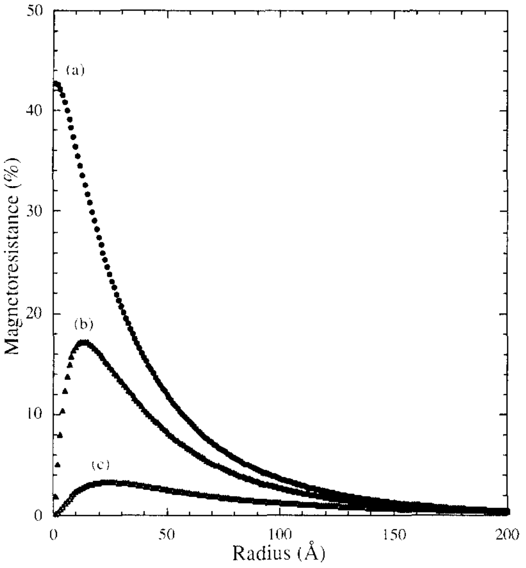

Fig. 1. Magnetoresistance ratio as a function of the radius of the granules for $l_1 = 150$ Å, $l_2 = 250$ Å, $l_3 = 15$ Å, $x = 0.3$, $p_S = 0$ and $\lambda_S = \infty$ (a), $12a_0$ (b), $2a_0$ (c).

Fig. 1 shows that the MR decreases with increasing radius of the particles and that the relevant ratio is (r_0/l_2) in contrast with what was claimed in Ref. [6]. Indeed, in Levy's approach of this model, in the absence of surface scattering, the magnetoresistance does not depend at all on the radius of the magnetic particles. In fact, it is only for small values of this ratio that the limit of a self-averaging system is obtained (see curve (a) in Fig. 1 for small values of r_0). In the case of large particles and large distances between them, the situation is close to the CIP geometry in multilayers. Then, only classical size-effects contribute to the magnetoresistance.

We next discuss the influence of electron scattering at the interfaces between the magnetic particles and the non-magnetic matrix. This interfacial scattering may be due to the roughness of the interfaces or to the difference of scattering potentials of impurities at the interface. The microscopic treatment of the problem of evaluating these potentials has been reported in Ref. [10], but there, only numerical calculations for finite clusters were peformed. It has been shown that the effective resistivity (and therefore the inverse of the electrons' mean-free path) in the matrix increases as the radius r_0 of the particles decreases. We approximate this dependence as follows:

$$l_{2\uparrow(\downarrow)}^{-1} = l_2^{-1} + 3x\frac{(1+p_S^2)a_0}{\lambda_S r_0} - (+)6x\frac{a_0 P_S}{\lambda_S r_0}L, \tag{11}$$

in which λ_S is the effective mean-free path associated with electron scattering at the interfaces, a_0 is the lattice constant, and P_S is a parameter describing the spin scattering asymmetry at the particles' interfaces. If one then introduces Eq. (11) into Eqs. (7)–(10), the influence of the surface scattering on the magnetoresistance will be taken into account. In Fig. 1, the MR is plotted versus r_0 for two values of λ_S and for $p_S = 0$ (no spin-dependent scattering at the interfaces). In Fig. 2, the same dependence is depicted for $p_S = 0.25$ and 0.52 (increasing interfacial spin-dependent scattering). This figure shows that the MR amplitude versus r_0 exhibits a maximum at an optimum value of r_0 which depends on the ratio (l_1/l_3) (characterizing the bulk spin-dependent scattering) and on the value of λ_S (influenced by the interfacial spin-dependent scattering). Again

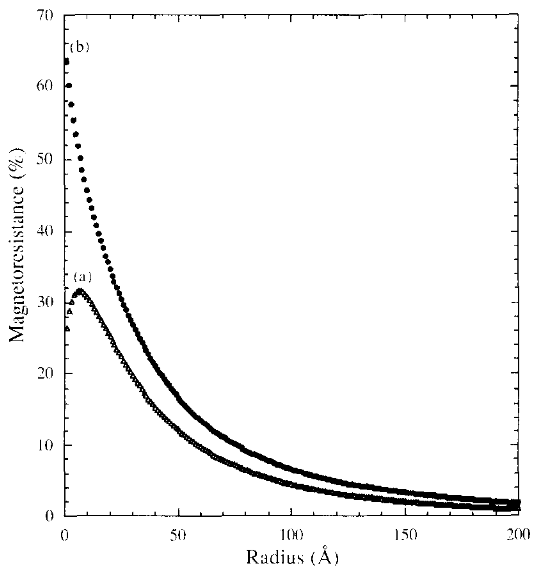

Fig. 2. Magnetoresistance ratio as a function of the radius of the granules for $l_1 = 150$ Å, $l_2 = 250$ Å, $l_3 = 15$ Å, $x = 0.3$, $\lambda_S = 12a_0$ and $p_S = 0.25$ (a), 0.52 (b).

the physical picture is clear: for large values of r_0, the second term in Eq. (11) can be neglected. The MR then decreases with increasing r_0 as $(l_2/r_0)^2 e^{-(2R/l_2)}$ due to the decrease in the influence of the classical size-effect. However, for small values of r_0, the effective electron mean-free path in the matrix decreases as r_0 decreases. As a result, if the surface scattering is spin-dependent ($p_S \neq 0$), the MR versus r_0 exhibits a maximum only if $p_S < p_B = (\sqrt{l_\uparrow} - \sqrt{l_\downarrow})/(\sqrt{l_\uparrow} + \sqrt{l_\downarrow})$. Otherwise, if $p_S > p_B$, the MR decreases monotonically with increasing r_0. Therefore, from experimental data it is possible to distinguish whether interfacial or bulk asymmetry is dominant. It is important to note that not only the relative MR ($\Delta\sigma/\sigma(L=1)$) shows a maximum as a function of r_0 but also the absolute ($\Delta\sigma$), as shown in Fig. 3.

It is interesting to compare our results with those obtained by Levy et al. [6,7]. They claimed that granular systems are self-averaging, so that for any values of r_0, the system can be described by only two mean-free paths $\bar{l}^\uparrow$ and $\bar{l}^\downarrow$, where $\bar{l}^{\uparrow(\downarrow)}$ are defined by (Eqs. (5), (12) and (13) in Ref. [6]):

$$(\bar{l}^{\uparrow(\downarrow)})^{-1} = \xi_0 \pm \xi_1 L, \tag{12}$$

with

$$\xi_0 = \frac{1-x}{l_2} + x\frac{l_1 + l_3}{2l_1 l_3} + 3x\frac{(1+p_S^2)a_0}{\lambda_S r_0} \tag{13}$$

and

$$\xi_1 = x\frac{l_1 - l_3}{2l_1 l_3} + \frac{6xa_0 p_S}{\lambda_S r_0}. \tag{14}$$

From Eqs. (12)–(14), the following relation has been obtained in Ref. [6]:

$$\frac{\Delta\rho}{\rho(H=0)} = \frac{\rho(H=0) - \rho(H)}{\rho(H=0)} = \frac{\Delta\sigma}{\sigma(L=1)} = \frac{\xi_1^2}{\xi_0^2}. \tag{15}$$

One may notice from Eqs. (12)–(15), it follows that for $(1/\lambda_S) \to 0$ surface scattering), $(\Delta\rho/\rho)$ does not depend on r_0. This seems rather unphysical. If one then takes into account surface scattering, according to Eqs. (12)–(15), $p_S > p_B = (\sqrt{l_1} - \sqrt{l_3})/(\sqrt{l_1} + \sqrt{l_3})$, $\Delta\sigma/\sigma(L=1)$ increases monotonically with decreasing r_0. In contrast, for our model which properly takes into account the finite size-effect, $\Delta\sigma/\sigma(L=1)$ shows a maximum versus r_0 (see Figs. 1 and 2), in agreement with many experimental observations in these heterogeneous alloys [11].

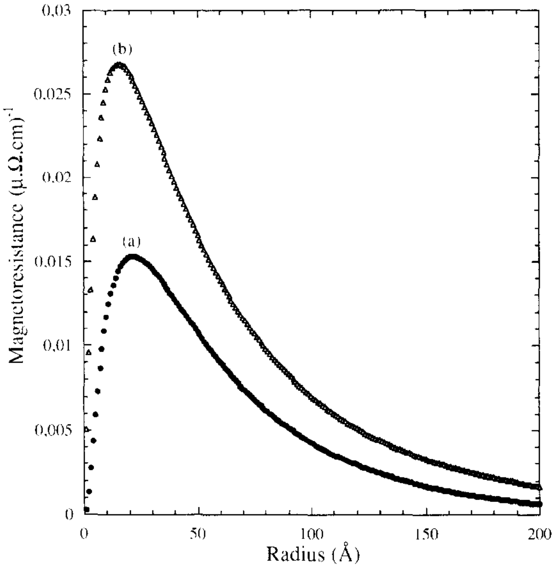

Fig. 3. Magnetoresistance as a function of the radius r_0 for $l_1 = 150$ Å, $l_2 = 250$ Å, $l_3 = 15$ Å, $x = 0.3$, $\lambda_S = 12a_0$ and (a) $p_S = 0$, (b) $p_S = 0.25$.

3. Conclusion

(1) Heterogeneous alloys are not completely 'self-averaging' systems. They can be considered as self-averaging only when the size of the particles and their distances are small as compared to all mean-free paths ($r_0/l_i \ll 1$). For larger ratios r_0/l_i, the classical spin-dependent size-effects determine the MR amplitude in analogy with the CIP (current in plane) geometry in magnetic multilayers. The difference with the CIP case is the self-averaging character of the surface scattering in heterogeneous systems.

(2) The existence of a maximum in the variation of $\Delta\sigma/\sigma(L=1)$ as a function of the radius r_0 and the optimum value of r_0 for which this maximum occurs, are results of the interplay of spin-dependent electron scattering in the bulk of the ferromagnetic particles and at their interfaces with the non-magnetic matrix.

(3) When the interfacial spin-dependent scattering asymmetry is not significantly larger than the bulk one, then the interfacial scattering may lead to a decrease of the absolute maximum of the magnetoresistance (one can compare the values of $\Delta\sigma/\sigma(L=1)$ without surface scattering and with surface scattering in Fig. 1).

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